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Parametric X-ray radiation at a small angle near the velocity direction of the relativistic particle

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Abstract

It is shown, that the angular distribution of parametric X-ray radiation, emitted near the forward direction is essentially narrower than that emitted at large angles. The analytical expressions are obtained for integral PXR intensity near the forward direction.

1. Introduction

As is well known the refraction of the electromagnetic field associated with the electron passing through the matter with a uniform velocity originates the Vavilov–Cherenkov and transition radiations. The Cherenkov emission of photons by a charged particle occurs whenever the index of refraction $n(\omega) > 1$ (ω is the photon frequency). For X-ray frequencies being higher than typical atom's ones the refractive index has an universal form

$$n(\omega) = 1 - \frac{\omega_L^2}{2\omega^2}, \tag{1}$$

where ω_L is the Langmuir frequency.

But as it was first shown in [1] that the effective refractive index of virtual photons of the electromagnetic field of a relativistic charged particle passing through a crystal can be anyone due to photon diffraction by a crystal.

As a result, this charged particle can emit Cherenkov X-rays. The classical [2–4], and quantum theories [5,6] of this radiation, called as parametric X-ray radiation (PXR), has been given. Later several theoretical papers were published on this problem [7,8].

According to [2,5,6] PXR can be described as diffraction of virtual photons of the electromagnetic field associated with a charged particle by a crystal planes. It is very important that only those photons can really be radiated for which the Cherenkov condition is fulfilled

$$1 - \beta n(\omega, \boldsymbol{k}) \cos \vartheta = 0, \tag{2}$$

where $n(\omega, \boldsymbol{k})$ is the refractive index of X-rays in a crystal, ϑ is the angle between the photon vector $\boldsymbol{k}$ and the particle velocity $\boldsymbol{v}$.

The unique properties of PXR such as tunability, high spectral brightness and possibility of emission at a large angle greatly exceeding a typical radiation angle of a relativistic particle γ^{-1} (γ is the Lorentz-factor of the particle) make it a potentially useful source of X-rays for wide variety of applications [6,9]. The predictions [2–6] for the spectral and angular features of PXR have been confirmed by [9–12].

However, the detailed analysis has been made for a large angle radiation only. As a result, it has been concluded in some papers that PXR is radiated only at a large angle [8,12]. According to [12] PXR is the well-known resonance radiation [13]. According to [12] there is a new "PXR of type B" for low energy electrons, which is not related to the Cherenkov effect. Note, however, that according to the theory of Baryshevsky and Feranchuk [5,6], each photon with the frequency ω, emitted at the large angle, corresponds to the photon with the same frequency ω, but emitted at a small angle $\vartheta \leq 1/\gamma$ near the forward direction (near velocity direction). The frequency of this photon does not depend on the particle energy. I would like to remind that the frequency of the photon emitted due to the "resonance radiation" mechanism, is proportional to γ^2.

It is very important to observe PXR at a small angle relative to the forward direction, since the PXR observation in this case will uniquely prove the quasi Cherenkov mechanism of PXR.

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In this paper we show that the angular distribution (AD) of PXR photons, emitted in the forward direction is sufficiently narrower than the AD of PXR beam emitted at large angles. The analytical expression has been also obtained for the integral PXR intensity in the forward direction. It is shown, that in this case the PXR intensity does not depend on a particle energy E, if $E > mc^2/\sqrt{n-1}$, where m is the particle mass (for X-rays $n-1 = 10^{-5}$–10^{-6}, that corresponds $E > 100$ MeV). In this case the number of photons per particle $N_0 \approx 10^{-5}$ and $N_0 \approx N_\tau$, where N_τ is the number of photons emitted at a large angle.

The PXR intensity falls off if the particle energy $E < mc^2/\sqrt{n-1}$. As a result, for the electron energy $E \approx 5$–10 MeV the number of photons $N_0 \approx 10^{-10}$–10^{-11} that is equal to the number of photons emitted at a large angle within the angle cone $\vartheta_D \sim \sqrt{n-1}$.

As is well known the spectral density of radiation per unit solid angle, denoted by $W_{n,\omega}$ (where $\boldsymbol{n} = \boldsymbol{k}/k$ and $\boldsymbol{k}$ and ω are the wave vector and the frequency of emitted quanta), can be expressed in terms of wave fields $\boldsymbol{E}(\boldsymbol{r}, \omega)$, produced by a charge at a large distance r from a crystal as follows:

$$W_{n,\omega} = \frac{cr^2}{4\pi} |\boldsymbol{E}(\boldsymbol{r}, \omega)|^2. \tag{3}$$

With the help of Eq. (3) the differential number of quanta registered by a detector is calculated as

$$\frac{d^2N}{d\omega\, d\Omega} = \frac{W_{n,\omega}}{\hbar\omega}.$$

In order to determine the field $\boldsymbol{E}(\boldsymbol{r}, \omega)$, one has to solve the Maxwell equations:

$$-\mathrm{curl}\boldsymbol{E}(\boldsymbol{r}, \omega) + \omega^2\hat{\varepsilon}(\boldsymbol{r}, \omega)\boldsymbol{E}(\boldsymbol{r}, \omega) = -\frac{4\pi i\omega}{c^2}\boldsymbol{j}(\boldsymbol{r}, \omega), \tag{4}$$

where $\hat{\varepsilon}(\boldsymbol{r}, \omega) = 1 + \hat{\chi}(\boldsymbol{r}, \omega)$ is the tensor of the crystal permitivity and $\hat{\chi}(\boldsymbol{r}, \omega)$ is the tensor of the crystal susceptibility, $\boldsymbol{j}(\boldsymbol{r}, \omega)$ is the Fourier component of current produced by a moving charge. The transverse solution of (4) can be found using the Green function satisfying the following equation:

$$\hat{G} = \hat{G}_0 + \hat{G}_0\frac{\omega^2}{4\pi}\hat{\chi}\hat{G}. \tag{5}$$

Here $\hat{G}_0$ is the transverse Green functions of Maxwell's equations with $\hat{\chi} = 0$.

$$\hat{G}_0(\boldsymbol{r}, \boldsymbol{r}', \omega) = \frac{\exp ik|\boldsymbol{r} - \boldsymbol{r}'|}{|\boldsymbol{r} - \boldsymbol{r}'|}\hat{I}, \tag{6}$$

where $\hat{I}$ is the unit tensor, $I_{i,l} = \Sigma_s e_i^s e_l^{s*}$, where $i, l = x, y, z$; $\boldsymbol{e}^s$ is the unit vector of the transverse polarization, $s = 1, 2$, $\boldsymbol{e}^s \perp \boldsymbol{k}$, $\boldsymbol{e}^1 \perp \boldsymbol{e}^2 \perp \boldsymbol{k}$.

With the help of $\hat{G}$ we can readily find the field of interest:

$$E_i(\boldsymbol{r}, \omega) = \frac{i\omega}{c^2}\int G_{il}(\boldsymbol{r}, \boldsymbol{r}', \omega)j_l(\boldsymbol{r}', \omega)d^3r'. \tag{7}$$

We are interested in the field at a large distance from a target. Taking $r \to \infty$, accounting for (5) and choosing the asymptotic form of $\hat{G}_0$ as

$$\lim_{r\to\infty} \hat{G}_0(\boldsymbol{r}, \boldsymbol{r}', \omega) = \frac{\exp(ikr)}{r}\exp(-\boldsymbol{k}\boldsymbol{r}')\hat{I}; \qquad \boldsymbol{k} = k\frac{\boldsymbol{r}}{r}, \tag{8}$$

we can write [14]:

$$\lim_{r\to\infty} \hat{G}_{i,l}(\boldsymbol{r}, \boldsymbol{r}', \omega) = \frac{\exp(ikr)}{r}\sum_s e_i^s E_{\boldsymbol{k},l}^{(-)s*}(\boldsymbol{r}', \omega), \tag{9}$$

where $\boldsymbol{E}_{\boldsymbol{k}}^{(-)s*}$ is the solution of homogeneous Maxwell's equations

$$\left[-\mathrm{curl\ curl}\ \boldsymbol{E}_{\boldsymbol{k}}^{(-)}(\boldsymbol{r}, \omega) + \frac{\omega^2}{c^2}\boldsymbol{E}_{\boldsymbol{k}}^{(-)}(\boldsymbol{r}, \omega)\right]_i + \chi_{i,j}^* E_{\boldsymbol{k},j}^{(-)}(\boldsymbol{r}, \omega) = 0. \tag{10}$$

If a wave is incident on a crystal with a finite thickness the solution of (10) can be presented as [14]:

$$\lim_{r\to\infty} \boldsymbol{E}_{\boldsymbol{k}}^{(-)s}(\boldsymbol{r}, \omega) = \boldsymbol{e}^s \exp(i\boldsymbol{k}\boldsymbol{r}) + \mathrm{const}\frac{\exp(-ikr)}{r}. \tag{11}$$

We can see that the solution $\boldsymbol{E}^{(-)}$ contains at infinity the incoming spherical wave.

The solution $\boldsymbol{E}_{\boldsymbol{k}}^{(-)s}$ is related to the ordinary solution $\boldsymbol{E}_{\boldsymbol{k}}^{(+)s}$ of homogeneous Maxwell's equations (4), describing a plane wave scattering by a target (crystal), in the following way: $\boldsymbol{E}_{\boldsymbol{k}}^{(-)s*} = \boldsymbol{E}_{\boldsymbol{k}}^{(+)s}$. Finally, with the help of Eq. (7) we obtained:

$$E_i(\boldsymbol{r}, \omega) = \frac{e^{ikr}}{r}\frac{i\omega}{c^2}\sum_s e_i^s \int \boldsymbol{E}_{-\boldsymbol{k}}^{(+)s}(\boldsymbol{r}', \omega)\boldsymbol{j}(\boldsymbol{r}', \omega)d^3r' = \frac{e^{ikr}}{r}\frac{i\omega}{c^2}\sum_s e_i^s \int \boldsymbol{E}_{\boldsymbol{k}}^{(-)s*}(\boldsymbol{r}', \omega)\boldsymbol{j}(\boldsymbol{r}', \omega)d^3r'. \tag{12}$$

The current $\boldsymbol{j}(\boldsymbol{r}, \omega)$ is expressed as

$$\boldsymbol{j}(\boldsymbol{r}, \omega) = \int \exp(i\omega t)\boldsymbol{j}(\boldsymbol{r}, t)\, dt = eQ\int \exp(i\omega t)\boldsymbol{v}(t)\delta(\boldsymbol{r} - \boldsymbol{r}(t))\, dt, \tag{13}$$

where eQ is the particle charge, $\boldsymbol{v}(t)$ is the particle velocity.

As a result, using Eq. (3) we can obtain an expression for the differential number of photons with the polarization vector $\boldsymbol{e}^s$ emitted in the $\boldsymbol{k}$ direction:

$$\frac{\mathrm{d}^2 N_s}{\mathrm{d}\omega\,\mathrm{d}\Omega} = \frac{e^2 Q^2 \omega}{4\pi^2 \hbar c^3} \left| \int_{-\infty}^{\infty} \boldsymbol{v} \boldsymbol{E}_{\boldsymbol{k}}^{(-)s}(\boldsymbol{r}(t), \omega) \exp(-\mathrm{i}\omega t)\, \mathrm{d}t \right|^2. \tag{14}$$

2. General expression for the PXR emission rate

Let a particle moving with an uniform velocity be incident on a crystal plate with the thickness L being $L \ll L_c$, where $L_c = (\omega q)^{-1/2}$ is the coherent length of bremsstrahlung $q = \overline{\theta}^2/4$ and $\overline{\theta}^2$ is the mean square angle of multiple scattering. The later requirement allows us to neglect the multiple scattering of particles on atoms. In the opposite case the theoretical method can be found in [4].

According to Eq. (14), in order to determine the number of quanta emitted by a particle passing through a crystal plate we should first find the explicit expressions for the solutions $\boldsymbol{E}_{\boldsymbol{k}}^{(-)s}$. As was mentioned above, the field $\boldsymbol{E}_{\boldsymbol{k}}^{(-)s}$ can be found from the relation $\boldsymbol{E}_{\boldsymbol{k}}^{(-)s} = (\boldsymbol{E}_{-\boldsymbol{k}}^{(+)s})^*$ if one knows the solution $\boldsymbol{E}_{\boldsymbol{k}}^{(+)s}$, describing the photon scattering by a crystal. The solution $\boldsymbol{E}_{\boldsymbol{k}}^{(-)s}$ can be obtained by straightforward application of Eq. (10) as well.

Making the Fourier transformation of these equations we can derive a set of equations matching the incident and diffracted waves. In the case of two strong waves excited under diffraction (so-called two-beam diffraction case [15]) we can obtain the following set of equations for determining wave amplitudes:

$$\begin{aligned} &\left(\frac{k^2}{\omega^2} - 1 - \chi_0^*\right) \boldsymbol{E}_{\boldsymbol{k}}^{(-)s} - c_s \chi_{-\tau}^* \boldsymbol{E}_{\boldsymbol{k}_\tau}^{(-)s} = 0, \\ &\left(\frac{k_\tau^2}{\omega^2} - 1 - \chi_0^*\right) \boldsymbol{E}_{\boldsymbol{k}_\tau}^{(-)s} - c_s \chi_\tau^* \boldsymbol{E}_{\boldsymbol{k}}^{(-)s} = 0. \end{aligned} \tag{15}$$

Here $\boldsymbol{k}_\tau = \boldsymbol{k} + \boldsymbol{\tau}$, $\boldsymbol{\tau}$ is the reciprocal lattice vector: χ_0, χ_τ are the Fourier components of the crystal susceptibility. It is well known that the crystal is described by a periodic susceptibility (see, for example, [15]):

$$\chi(\boldsymbol{r}) = \sum_{\tau} \chi_\tau \exp(\mathrm{i}\boldsymbol{\tau}\boldsymbol{r}). \tag{16}$$

$c_s = \boldsymbol{e}^s \boldsymbol{e}_\tau^s$, $\boldsymbol{e}^s(\boldsymbol{e}_\tau^s)$ are the unit polarization vectors of the incident and diffracted waves, respectively.

The condition for the linear system (15) to be solvable leads to a dispersion equation that determines the possible wave vectors $\boldsymbol{k}$ in a crystal. These wave vectors are convenient to be presented in the form:

$$\boldsymbol{k}_{\mu s} = \boldsymbol{k} + \varkappa_{\mu s}^* \boldsymbol{N}, \qquad \varkappa_{\mu s} = \frac{\omega}{c\gamma_0} \varepsilon_{\mu s}^*,$$

where $\mu = 1, 2$; $\boldsymbol{N}$ is the unit vector of a normal to the entrance crystal surface which is directed into a crystal,

$$\begin{aligned} \varepsilon_{1(2)s} = {} & \tfrac{1}{4}\left[(1+\beta_1)\chi_0 - \beta_1 \alpha_{\mathrm{B}}\right] \\ & \pm \tfrac{1}{4}\Big\{\big[\underbrace{(1+\beta_1)\chi_0 - \beta_1 \alpha_{\mathrm{B}} - 2}_{= (1-\beta_1)\chi_0 + \beta_1 \alpha_{\mathrm{B}}}\chi_0\big]^2 \\ & + 4\beta_1 C_S^2 \chi_\tau \chi_{-\tau}\Big\}^{1/2}, \end{aligned} \tag{17}$$

$\alpha_{\mathrm{B}} = (2\boldsymbol{k}\boldsymbol{\tau} + \tau^2)k^{-2}$ is the off-Bragg parameter ($\alpha_{\mathrm{B}} = 0$ if the exact Bragg condition of diffraction is fulfilled),

$$\gamma_0 = \boldsymbol{n}_\gamma \cdot \boldsymbol{N}, \quad \boldsymbol{n}_\gamma = \frac{\boldsymbol{k}}{k}, \quad \beta_1 = \frac{\gamma_0}{\gamma_1}, \quad \gamma_1 = \boldsymbol{n}_{\gamma\tau} \cdot \boldsymbol{N},$$

$$\boldsymbol{n}_{\gamma\tau} = \frac{\boldsymbol{k} + \boldsymbol{\tau}}{|\boldsymbol{k} + \boldsymbol{\tau}|}.$$

The general solution of Eqs. (10), (15) inside a crystal is:

$$\boldsymbol{E}_{\boldsymbol{k}}^{(-)s}(\boldsymbol{r}) = \sum_{\mu=1}^{2} \left[\boldsymbol{e}^s A_\mu \exp(\mathrm{i}\boldsymbol{k}_{\mu s}\boldsymbol{r}) + \boldsymbol{e}_\tau^s A_{\tau\mu} \exp(\mathrm{i}\boldsymbol{k}_{\mu s\tau}\boldsymbol{r}) \right]. \tag{18}$$

By matching these solutions with the solutions of Maxwell's equation for the vacuum area we can find the explicit form of $\boldsymbol{E}_{\boldsymbol{k}}^{(-)s}(\boldsymbol{r})$ throughout the space. It is possible to discriminate several types of diffraction geometries, namely, the Laue (a) and the Bragg (b) schemes are the most well-known.

(a) Let us consider the PXR in the Laue case.

In this case, the electromagnetic waves emitted by a particle both in the forward and diffracted directions leave the crystal through the same its surface ($k_z > 0$, $k_z + \tau_z > 0$, the axis Z is parallel to the normal N (where N is the normal to the crystal surface being directed inside a crystal). By matching the solutions of Maxwell's equations on the crystal surfaces with the help of Eqs. (15,17,18) one can obtain the following expression for the Laue case:

$$\begin{aligned} \boldsymbol{E}_{\boldsymbol{k}}^{(-)s} = {} & \left\{ \boldsymbol{e}^s \left[-\sum_{\mu=1}^{2} \xi_{\mu s}^{0*} \mathrm{e}^{-\mathrm{i}\frac{\omega}{\gamma_0}\varepsilon_{\mu s}^* L} \right] \mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{r}} + \mathrm{e}_\tau^s \beta_1 \left[\sum_{\mu=1}^{2} \xi_{\mu s}^{\tau *} \mathrm{e}^{-\mathrm{i}\frac{\omega}{\gamma_0}\varepsilon_{\mu s}^* L} \right] \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\tau \boldsymbol{r}} \right\} \theta(-z) \\ & + \left\{ \boldsymbol{e}^s \left[-\sum_{\mu=1}^{2} \xi_{\mu s}^{0*} \mathrm{e}^{-\mathrm{i}\frac{\omega}{\gamma_0}\varepsilon_{\mu s}^*(L-z)} \right] \mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{r}} + \boldsymbol{e}_\tau^s \beta_1 \left[\sum_{\mu=1}^{2} \xi_{\mu s}^{\tau *} \mathrm{e}^{-\mathrm{i}\frac{\omega}{\gamma_0}\varepsilon_{\mu s}^*(L-z)} \right] \mathrm{e}^{\mathrm{i}\boldsymbol{k}_\tau \boldsymbol{r}} \right\} \\ & \times \theta(L-z)\theta(z) + \boldsymbol{e}^s \mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{r}} \theta(z-L), \end{aligned} \tag{19}$$

$-i\omega$ t

Если χ_0 зависит от поляризации, то достаточно в ф-х заменить поставить χ_0^S, χ_τ^S (это сейчас имеется в виду!) |||||

where

$$\xi^0_{1,2s} = \mp \frac{2\varepsilon_{2,1s} - \chi_0}{2(\varepsilon_{2s} - \varepsilon_{1s})};$$

$$\xi^\tau_{1,2s} = \mp \frac{c_s \chi_{-\tau}}{2(\varepsilon_{2s} - \varepsilon_{1s})}. \tag{20}$$

$\theta(z) = 1$ if $z \geq 0$ and $\theta(z) = 0$ if $z < 0$.

The substitution of Eq. (19) into (14) gives, for the Laue case the differential number of quanta of the forward directed parametric X-rays with the polarization vector e_s:

$$\frac{d^2 N^L_{0s}}{d\omega\, d\Omega} = \frac{e^2 Q^2 \omega}{4\pi^2 \hbar c^3} (e^s v)^2 \left| \sum_{\mu=1,2} \xi^0_{\mu s} e^{i\frac{\omega}{c\gamma_0}\varepsilon_{\mu s} L} \left[\frac{1}{\omega - kv} - \frac{1}{\omega - k^*_{\mu s} v} \right] \left[e^{i(\omega - k^*_{\mu s} v)T} - 1 \right] \right|^2, \tag{21}$$

where $T = L/c\gamma_0$ is the particle time of flight; $e_1 \| [k\tau]$; $e_2 \| [k e_1]$.

We can see that the formula (21) looks like the formula which describes the spectral and angular distribution of the Cherenkov and transition radiations in the matter with the refraction index $n_{\mu s} = k_{z\mu s}/k_z = 1 + \varkappa_{\mu s}/k_z$.

The spectral angular distribution for fotons in the diffraction direction $k_\tau = k + \tau$ can be obtained from (21) by the simple substitution

$$e_s \to e_{s\tau}, \qquad \xi^0_{\mu s} \to \beta_1 \xi^\tau_{\mu s},$$

$$\xi^\tau_{1(2)s} = \pm \frac{\chi_\tau c_s}{2(\varepsilon_{1s} - \varepsilon_{2s})}$$

$$k \to k_\tau, \qquad k_{\mu s} \to k_{\tau\mu s} = k_{\mu s} + \tau.$$

(b) Now let us consider PXR in the Bragg case. In this case, side by side with electro-magnetic wave emitted in the forward direction, an electromagnetic wave emitted by a charged particle in the diffracted direction and leaving the crystal through the surface of the particle entrance can be observed. By matching the solutions of Maxwell's equations on the crystal surface with the help of Eqs. (15), (17) and (18), we can get the formulae for the Bragg diffraction schemes.

It is interesting that the spectral angular distribution for photons emitted in the forward direction can be obtained from (21) by the following substitution, $\xi^0_{\mu s} \to \gamma^0_{\mu s}$,

$$\gamma^0_{1(2)s} = \left[2\varepsilon_{2(1)s} - \chi_0\right] \times \left[(2\varepsilon_{2(1)s} - \chi_0) - (2\varepsilon_{1(2)s} - \chi_0) \times \exp\left[i\frac{\omega}{\gamma_0}(\varepsilon_{2(1)s} - \varepsilon_{1(2)s})L \right] \right]^{-1}. \tag{22}$$

The spectral angular distribution of photons emitted in the diffracted direction can be obtained from (21) by the substitution

$$e_s \to e_{s\tau}, \quad k \to k_\tau, \quad k_{\mu s} \to k_{\mu\tau s},$$

$$\xi^0_{\mu s} \exp\left[i\frac{\omega}{\gamma_0}\varepsilon_{\mu s} L \right] \to \gamma^\tau_{\mu s},$$

where

$$\gamma^\tau_{1(2)s} = -\beta_1 [c_s \chi_\tau] \left[(2\varepsilon_{2(1)s} - \chi_0) - (2\varepsilon_{1(2)s} - \chi_0) \times \exp\left[i\frac{\omega}{\gamma_0}(\varepsilon_{2(1)s} - \varepsilon_{1(2)s})L \right] \right]^{-1}.$$

3. PRX intensity in the forward direction

Let us investigate PXR in the forward direction.

First of all we would like to note, that $\chi_0 < 0$ and we can get from Eq. (7) that only one root ε_{1s} gives the refractive index $n > 0$ for the Laue case. As a result, the difference $\omega - k_{1s\upsilon} v$ can now be zero and the term in (21), containing this difference in its denominator, will grow proportionally to L. On the other hand, the term in (21), containing the same difference $\omega - k_{2s} v$ but with the root ε_{2s} in its denominator, will not be linear to L because $\omega - k_{2s} v$ can never be zero. It means that the term containing the difference $\omega - k_{1s} v$ gives the principal contribution to the radiation intensity if the length $L_0 = L/\gamma_0 \gg l$ (where the vacuum coherent length $l = \lambda\gamma^2$, λ is wave length of the photon). It gives $L_0 \gg 10^{-2}$ cm for $\lambda = 10^{-8}$ cm and $\gamma = 10^3$. However, we must take into consideration all terms in (21) if $L_0 \sim l$. So, the radiation intensity will oscillate as a function of L with the space period of $l_0 = c/[\omega(\varepsilon_{1s} - \varepsilon_{2s})]$.

Let $L_0 \gg l$. From (21) we have

$$\frac{d^2 N_{0s}}{d\omega\, d\Omega} = \frac{e^2 Q^2}{\pi \hbar c^3} (e^s v)^2 \left| \frac{2\varepsilon_{2s} - \chi_0}{2(\varepsilon_{1s} - \varepsilon_{2s})} \right|^2 \times T\delta\left(\frac{1}{\gamma^2} + \vartheta^2 - 2\varepsilon_{1s} \right) \tag{23}$$

and the angular distribution is expressed as:

$$\frac{dN_{0s}}{d\Omega} = \frac{e^2 Q^2}{\pi \hbar c} \vartheta^2 \begin{bmatrix} \sin^2\varphi \\ \cos^2\varphi \end{bmatrix} \times \frac{\beta_1 r_s^2}{(\gamma^{-2} + \vartheta^2 - \chi_0)^2 \left[(\gamma^{-2} + \vartheta^2 - \chi_0)^2 + \beta_1 r_s \right]} \times \frac{\omega_B T}{\sin^2 \vartheta_B}, \tag{24}$$

where $\omega_B = \tau c/2 \sin \vartheta_B$ is the frequency of PXR radiation for which $\alpha_B = 0$; $r_s = c_s \chi_\tau \chi_{-\tau}$; ϑ_B is the Bragg angle, φ is the azimuth angle between the diffraction plane and the radiation plane formed by the vectors $\boldsymbol{k}$ and $\boldsymbol{v}$ $\sin^2\varphi$ refers to the photons with the polarization vector $\boldsymbol{e}_1 \| [\boldsymbol{k\tau}]$; $\cos^2\varphi$ refers to the photons with the polarization vector $\boldsymbol{e}_2 \| [\boldsymbol{k e}_1]$, $c_s = \cos 2\vartheta_B$.

According to Eq. (24) the photon angular distribution in the forward direction is proportional to $\vartheta^{-5}\mathrm{d}\vartheta$ for $\vartheta^2 \gg \gamma^{-2} + |\chi_0|$ and decreases quicker than that for the photons emitted in the diffraction direction, which is proportional to $\vartheta^{-1}\mathrm{d}\vartheta$.

For the total number of photons emitted in the forward direction we can obtain from (24):

$$N_{0s} = \frac{e^2Q^2}{8\hbar c}\,\frac{c_s|\chi_\tau\chi_{-\tau}|}{\sin^2\vartheta_B}\,\omega_B T\left\{\ln\frac{\sqrt{(\gamma^{-2}-\chi_0)^2+\beta_1 r_s}}{\gamma^{-2}-\chi_0} + \frac{\gamma^{-2}-\chi_0}{\sqrt{\beta_1 r_s}}\left(\frac{\pi}{2}-\operatorname{arctg}\frac{\gamma^{-2}-\chi_0}{\sqrt{\beta_1 r_s}}\right)-1\right\}. \quad (25)$$

According to (25) the number of photons N_{0s} does not depend on the particle energy if $\gamma^{-2} < |\chi_0|$ but the number of photons N_{0s} depends on the energy very sharply for $\gamma^{-2} > |\chi_0|$.

In the case of low energy we can obtain from (12):

$$N_{0s} \simeq \frac{e^2Q^2}{48\hbar c}\,\frac{\beta_1(c_s\chi_\tau\chi_{-\tau})^2}{\gamma^{-4}\sin^2\vartheta_B}\,\omega_B T \sim \gamma^4. \quad (26)$$

From Eq. (25) we can obtain $N_{0s} = 10^{-5}$ for $\chi_\tau = 10^{-5}$, $k_B = \omega_B/c = 10^9\ \mathrm{cm}^{-1}$, $L_0 = 10^{-1}$ cm, $T = L_0/c$, $\gamma^{-2} < |\chi_0|$, and $E = mc^2\gamma > mc^2/\sqrt{|\chi_0|} \geq 100$ MeV.

However, for $E = 10$ MeV, $\beta_1 = 10$ we have $\gamma^{-2} > |\chi_0|$ and from (26) we can obtain $N_{0s} \simeq 10^{-11}$.

Now let us briefly consider the Bragg scheme. The angular distribution for the photons emitted at large angles has been derived in [4]. From Eqs. (21),(22) we can obtain the angular distribution for the photons emitted in the forward direction:

$$\begin{aligned}\mathrm{d}N_{0s}^{B} = {} & \frac{e^2Q^2}{4\hbar c}|\beta_1||r_s|^2 \\ & \times\left|\left\{(\gamma^{-2}+\vartheta^2-\chi_0)^2-|\beta_1|r_s \right.\right. \\ & \left.\left. \times\exp\left[-\mathrm{i}\frac{\omega_B}{2\gamma_0 c}\,\frac{(\gamma^{-2}+\vartheta^2-\chi_0)^2-|\beta_1|r_s}{\gamma^{-2}+\vartheta^2-\chi_0}L\right]\right\}\right|^{-2} \\ & \times\left|\frac{(\gamma^{-2}+\vartheta^2-\chi_0)^2-|\beta_1|r_s}{(\gamma^{-2}+\vartheta^2-\chi_0)^2}\right|\frac{\omega_B T}{\sin^2\vartheta_B}\vartheta^3\,\mathrm{d}\vartheta. \quad (27)\end{aligned}$$

According to Eq. (27) the PXR angular distribution for this case oscillates as a function of ϑ, L, ω_B. If $\vartheta^2 \gg \gamma^{-2}$, χ_0 the oscillation period is $\vartheta_{0s} = \sqrt{c/\omega_B L_0}$. For $k_B = (\omega_B/c) = 10^9\ \mathrm{cm}^{-1}$, $L_0 = L/\gamma_0 = 10^{-2}$ cm, we have $\vartheta_{0s} = 3 \times 10^{-4}$.

For low energy electrons oscillations in N_{0s}^{B} disappear. As a result, the formula (27) turns to the formula (26) for the Laue scheme. It is very important to have in mind that PXR radiation in the forward direction cannot be obtained in the kinematical theory applied in [8,12]. So, the physics mechanism of PXR is the same as the quasi-Cherenkov mechanism of radiation both for the high and the low energies. Let us notice that the PXR photon number is proportional to Q^2. As a result, the PXR intensity is very high for heavy nuclei.

For example, for Pb the photon number may be 1 per a nucleus for $L = 1$ cm. It can be used for the detection of particles and for the measurement of their energy with a high accuracy.

Let us discuss the opportunity of experimental observation of PXR in the forward direction. As was mentioned above, for high particles energies ($E \geq mc^2/\sqrt{|\chi_0|} > 100$ MeV) the number of PXR quanta emitted in the forward direction is of the same order of magnitude as that in the diffraction direction. It is now well-known from both the theory and experimental data [4,6,9–12] that the number of bremsstrahlung quanta in the vicinity of the peak PXR emission essentially exceeds the number of bremsstrahlung quanta for the crystal target with the thickness considered by us. Consequently, the bremsstrahlung will not interfere with an observation of PXR in the forward direction. Only X-ray transition radiation can make difficulties for experimental observation of PXR. However, taking into account the fact that the transition radiation possesses the spectrum of $(\mathrm{d}N/\mathrm{d}\omega) \sim (\mathrm{d}\omega/\omega)$ [13] it is easy to obtain [6] that its intensity in the vicinity of the PXR spectral peak is lower than the PXR intensity as well (this spectral region is $\Delta\omega/\omega \sim \sqrt{\gamma^{-2} + |\chi_0|}$ [6]).

So, for observation of PXR in the forward direction it is necessary to use detectors with a high spectral resolution, for example, a semiconductor detector or a crystal diffractometer. It permits us to avoid a possible interference of transition radiation or bremsstrahlung.

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